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SOME MEASUREMENTS OF TURBULENT HEAT TRANSFER IN THE THERMAL ENTRANCE REGION OF CONCENTRIC ANNULI

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Abstract—Results are presented of measurements of turbulent heat transfer from the core tube of a concentric horizontal annulus. The radius ratios investigated are 2·88, 5·76 and 9·14 with a Reynolds number range of 5000–270000. The investigation covers both the fully developed situation and the thermal entrance region. The velocity profile was fully developed at the start of heating.

Some results are given for the variation of thermal entrance length with Reynolds number and radius ratio.

NOMENCLATURE

A, cross-sectional area;
b, radius ratio r_0/r_i;
B, temperature parameter;
C_p, specific heat;
d, diameter;
D, hydraulic diameter $2(r_0 - r_i)$;
f, friction factor;
h, heat-transfer coefficient;
i, current;
k, thermal conductivity;
Nu, Nusselt number hD/k_b;
Pr, Prandtl number $\mu C_p/k$;
q, heat flux per unit area;
Q, total heat flux;
r, radius;
Re, Reynolds number $(u_b/\nu)D$;
t, gas temperature;
T, absolute gas temperature;
u, velocity;
V, voltage;
x, axial co-ordinate;
x^+, non-dimensional axial co-ordinate x/D.

Subscripts
b, bulk;
c, core;
e, thermal entrance length;
i, inner;
o, outer;
x, at position x;
fd, fully developed;
w, wall.

Greek symbols
ρ, density;
μ, viscosity;
ν, μ/ρ;
τ, shear stress.

INTRODUCTION

THE PROBLEM of heat transfer in annular passages is of considerable technical importance. It has received the attention of many investigators in the last thirty years. However in common with other flow passages much of the early work was done without a sufficiently careful control of the boundary conditions which would be necessary for a systematic study. Thus, for example, Monrad and Pelton [1] and Foust and Christian [2] present results in disagreement by 100 per cent or so. A very comprehensive survey of the literature on this problem is given by Leung, Kays and Reynolds [3].

This publication also contains, amongst other work, extensive experimental results for heat transfer in concentric annuli which have clearly been obtained with considerable care. Results are presented for heating from both core and outer tubes with uniform heat flux for radius ratios 2, 2·67, 3·92 and 5·21. The Reynolds-number range was from 10000 up to 160000 with air as a working fluid. Measurements in the thermal entrance region were taken in order to obtain the fully developed results by extrapolation. This result was calculated by fitting an exponential function to a set of measurements in the entrance region and extra polating. Lanczos [4] has discussed the possible unsatisfactory results arising from such a procedure. Also since the annulus was mounted vertically it is surprising that such a careful study made no mention of possible natural convection effects.

More recently, Lee and Barrow [5] have given results for fully developed heat transfer from a uniformly heated core tube. The radius ratios studied were 1·632, 2·583 and 3·875 with air as the working fluid and a Reynolds-number range of 10000 to 50000.

The results of these authors disagree with reference [3] by as much as 20 per cent. This is shown in Fig. 1. The annulus in this case was mounted horizontally. The velocity profile however seems to have been developing along with the temperature profile since there was apparently no annular section upstream of the heating section.

The purpose of this work is to present experimental results for turbulent heat transfer from the uniformly heated core tube of a concentric annulus including both the thermal entrance region and the fully developed situation.

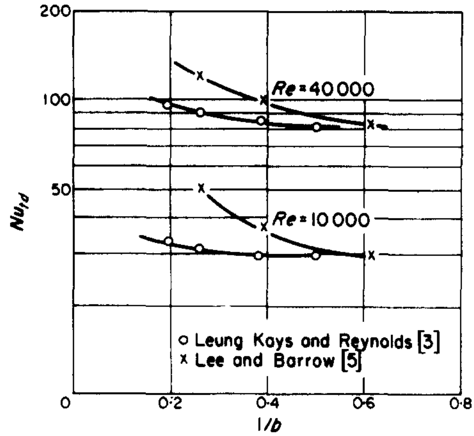

FIG. 1. Comparison of two recent results.

APPARATUS

The apparatus is shown in Fig. 2. The metering section was a plain Perspex tube and two sizes were used; 4-in i.d. and 1·75-in i.d. The length was 6 ft. A pitot tube traversing gear was fitted 9 in from the downstream end.

The annular section consisted of 10·5 ft of unheated tube followed by 9·5 ft in which the core tube was heated electrically. The outer tube was of solid drawn brass 3-in o.d. and 2·88-in i.d. It was joined at the start of the heating section by a carefully made spigot flange.

The stainless steel core tubes were 1·000-in o.d. and 0·920-in i.d. 0·500-in o.d. and 0·360-in i.d. and 0·315-in o.d. and 0·162-in i.d. They were supported concentrically in the outer tube by aerofoil shaped struts $\frac{1}{8}$-in thick in the heating section and by plain round struts $\frac{1}{8}$-in in diameter elsewhere. The sag of the core tube was nowhere greater than 3 per cent of the difference in radii of the tubes. All the supports could be adjusted radially and those at the start of the

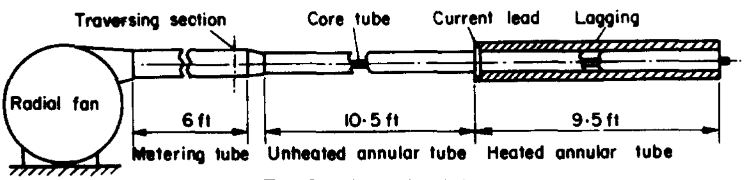

FIG. 2. Schematic of rig.

heating section had a brass ring soldered to their ends concentric with the outer tube and were used as current leads. The upstream core was insulated from the downstream core as were all the struts in the heating section. The arrangement is shown in Fig. 3.

through a variac and transformer. The heating could be controlled quite accurately. The current available was up to 250 V and this was measured through a 60:1 current transformer and suitable ammeter.

In order to calculate the local Nusselt number

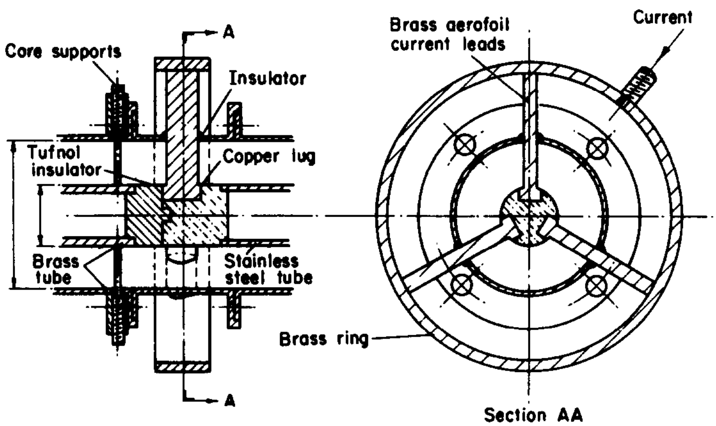

FIG. 3. Core support and current feed.

The outer tube was insulated by shaped polystyrene foam cladding 2-in thick. Three sizes of radial fan were used to give a wide range of Reynolds number.

FLOW, POWER AND TEMPERATURE MEASUREMENT

The volume flow of air was measured in each case by traversing the metering tube with a pitot tube of 0·040-in o.d. and 0·026-in i.d. using a micromanometer. The traversing interval was adjusted to give a constant manometer deflection of about 0·050 in. On the average some 20 readings were taken along with the radius for each flow. These traverses gave a valuable check on the symmetry of the flow entering the annulus. In all cases for which results are presented here this was found to be perfectly satisfactory.

The larger metering tube was used for high flow rates and the smaller for low flow rates so that the pitot tube was not subject to compressibility or low Reynolds-number effects.

The power supply was from 220 V mains it is clearly necessary to know the local heat flux and the bulk temperature. Both these require a knowledge of the heat generation along the tube or in other words the voltage drop along the tube together with the current.

In the 1-in core tube temperature and voltage were measured fairly easily. Voltage tappings and thermocouples were inserted into the small holes in the core tube wall and soldered in place. They were then emeried flush. The two other cores were too small to allow this procedure. Accordingly a travelling thermocouple was made for each. These consisted of a suitably shaped spring wire attached to a flexible Bowden cable whose outer diameter was close to the inner diameter of the core tube. This is shown in Fig. 4. The thermocouples were embedded in small blobs of solder at the ends of the spring wire and were thus pressed quite firmly in contact with the inner surface of the core tube.

The thermocouples themselves were used as voltage tappings in this case and a cathode ray oscilloscope was used to measure the

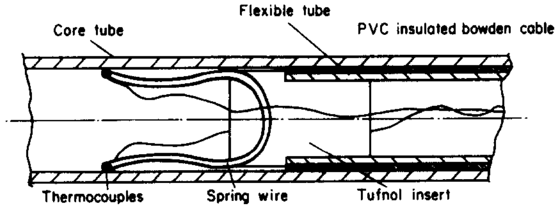

FIG. 4. Travelling thermocouple arrangement.

voltage since the resistance of the thermocouple wires was significant.

The thermocouples were 30 s.w.g. glass insulated copper–constantan referenced to a single cold junction of ice and water through a multiselector switch.

In the 1-in core the thermocouples were at 2-in intervals over the first 2 ft then at 4 in for 1 ft and over the remainder at 1-ft intervals. There were 22 thermocouples in all. The voltage tappings were similar. With the travelling thermocouples measurements could be taken anywhere.

Ten thermocouples were evenly spaced along the heating section on the outer surface of the outer tube and three each were placed in the air stream at the beginning and end of the heated section. The air stream thermocouples were spaced at 120°.

Thermocouple potentials were measured by a Cambridge potentiometer reading to 0·01 mV.

DATA REDUCTION AND ACCURACY

It is considered that the volume flow measurement has zero error on its own merit since there is no better method against which it might be checked. The ammeter and voltmeter were checked and the accuracy of power measurement is considered to be $\pm 1{\cdot}5$ per cent. Since the C.R.O. has a resistance of megohms the contact resistance and hence voltage drop across the travelling voltage tap is considered negligible.

In reference [3], Leung, Kays and Reynolds found that radiation from the core to the outer tube was significant and further that small heat fluxes were present between the outer tube and the gas stream.

In this work the heating rates were controlled to keep the temperatures involved as low as possible. It was found that the temperature difference across the polystyrene insulation during the experiments was only 15–25 degF and the heat leak thus calculated was negligible compared with the heat flux from the core tube. On calculating the radiation loss to the outer tube this also was found to be negligible. As a final check the bulk temperature rise calculated from the heat generated and allowing for axial conduction agreed within ± 4 per cent with that measured by averaging the three air-stream thermocouples at the beginning and end of the heated section. The main purpose of the three air-stream thermocouples at the end of the heated section was to check the symmetry of the temperature field. This was quite satisfactory.

Using the values to check the calculated bulk temperature rise involves some error since they are almost certainly different to the value which would be given by a mixing chamber. Accordingly agreement to ± 4 per cent is probably rather fortuitous.

From a heat balance on an element, dx, of core tube it is easily shown that the heat flux per unit area q_x is given by

$$q_x = \frac{k_c A_c (\mathrm{d}^2 t/\mathrm{d}x^2)_x - i\,\mathrm{d}V/\mathrm{d}x}{\pi d_c}. \tag{1}$$

The conduction term can be found by graphically differentiating the core temperature profile which is a dubious process. Fortunately, however, since A_c is small this term is not very large compared with the first and away from the very beginning can be neglected altogether. Since the temperature differences along the core tube are not too large the variation of resistance is negligible and hence $\mathrm{d}V/\mathrm{d}x$ is constant.

A condition of uniform heat flux is attained.

On integrating equation (1) the total heat generated, Q_x from $x = 0$ up to the point x is given by

$$Q_x = \frac{k_c A_c}{\pi d_c}\left[\left(\frac{dt}{dx}\right)_x - \left(\frac{dt}{dx}\right)_o\right] - i\left[\frac{V_x - V_o}{\pi d_c}\right]. \quad (2)$$

In equation (2) whether or not conduction can be neglected depends on the relative magnitudes of the terms. The first term is small compared with the second for all values of x because A_c is small and also because as the difference between the temperature gradients becomes large so does $V_x - V_o$.

Thus, it is justifiable to consider the axial variation of bulk temperature to be linear up to quite close to the start of heating.

The possible difference between the temperatures measured by the travelling thermocouple at the inner surface of the core tube and that obtaining at the outer surface was calculated and found to be negligible.

The Nusselt number at position x, Nu_x was calculated from the definition

$$Nu_x = \frac{h_x D}{k_b}$$

i.e.

$$Nu_x = \frac{q_x}{(t_c - t_b)_x}\frac{D}{k_b} \quad (3)$$

where q_x was calculated from equation (1) and t_c was taken from the graphs of core temperature and t_b from equation (2) plotted on to the same graphs.

The error estimates for the Reynolds and Nusselt numbers mainly depend on the error estimates of the temperature and electrical measurements. The linear dimension and pressure measurements can be made with very small errors. It was estimated that the Reynolds number was correct to ± 2 per cent and the Nusselt number to ± 3 per cent. This is about the same as reference [3].

EXPERIMENTAL RESULTS

It was established in the beginning that the velocity profile at the start of heating was fully developed. The profile was considered fully developed when the measured static pressure along the annulus was linear, allowing for slight disturbances due to the supports.

The radius ratios investigated were 2·88, 5·76 and 9·14. The non-dimensional unheated length for each was thus $x^+ = 67$, 53 and 50 respectively. This is much less than the 250 equivalent diameters recorded by Rothfus *et al.* [6] but adequate according to more recent results of Brighton and Jones [7]. The hydrodynamic development lengths provided in reference [3] were less than in this work being 48, 38, 16·5 and 14·7. The heated lengths in this work were 60, 48 and 45 equivalent diameters. These were considered sufficient.

The Reynolds number range attained differed according to the size of core tube. Thus for $b = 2·88$ experiments were carried out in the range $5000 < Re < 180000$; for $b = 5·76$, $12500 < Re < 240000$ and for $b = 9·14$, $8300 < Re < 276000$. The heat flux per unit area was varied from 400 to 2500 Btu/ft^2/h/degF according to the mass flow.

The bulk temperature rise was 15–30 degF and the air temperature at entry to the heated section about 10–25 degF above ambient in most cases. This gave a bulk temperature–wall temperature difference of about 50–130 degF at the end of the heated section. A typical result of temperature measurements is shown in Fig. 5.

It was considered that the core temperature gradient over the latter part of the heating section was sufficiently linear and parallel to the bulk temperature to justify assuming that heat transfer was fully developed. The radius ratio 2·88 was investigated first and the results of 34 tests are presented here. Since these results seemed to indicate that no significant errors were present, the other 2 annuli were investigated with fewer tests. Seventeen results are given for $b = 5·76$ and 24 for $b = 9·14$.

The fully developed Nusselt number, Nu_{fd}, is

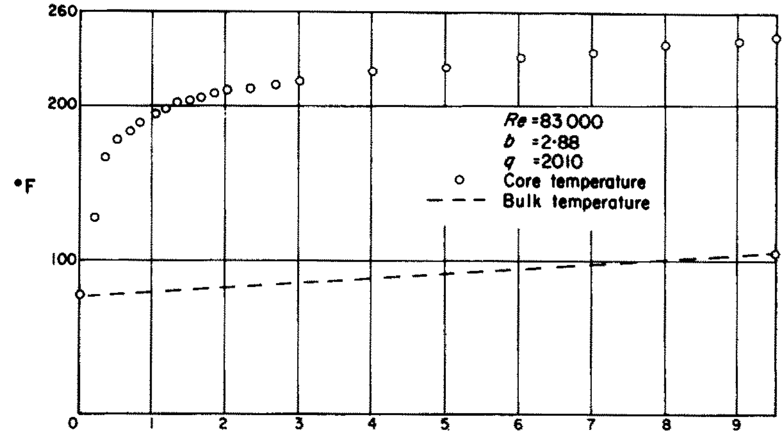

FIG. 5. Core and bulk-temperature variation.

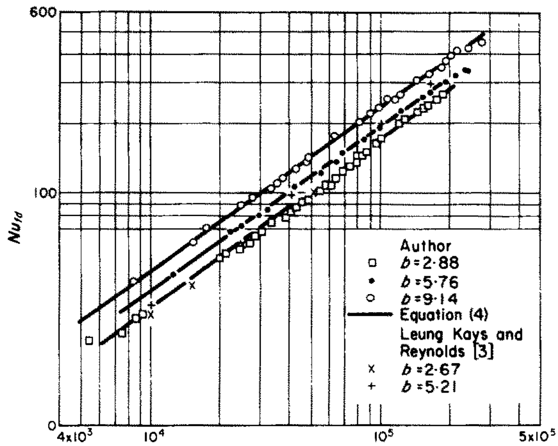

FIG. 6. Fully developed heat-transfer results.

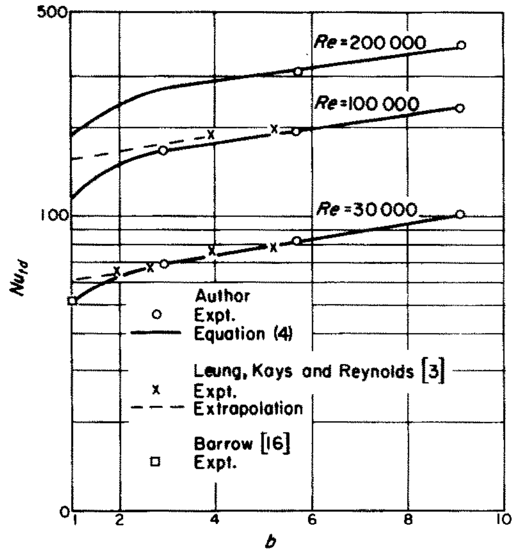

FIG. 7. Effect of radius ratio on fully developed heat transfer.

taken as the average of the last 3 readings along the heated section and is plotted in Fig. 6 against Reynolds number. Some appropriate values from reference [3] are included for comparison. A further comparison between this work and reference [3] is given in Fig. 7 where Nu_{fd} is plotted against radius ratio for Re = 30000, 100000 and 200000.

Nu_x, in the entrance region was calculated from equation (3) and results for a representative selection of the Reynolds numbers are given in Figs. 8–10 for each radius ratio. A comparison is made with some of the entrance region results of Leung, Kays and Reynolds [3] in Figs. 7 and 9. These results were given as a non-dimensional temperature not as a Nusselt number. Calculating the Nu_x was a rather dubious process because of the small scale of the graphs, especially for the higher Reynolds numbers. However, the results are in reasonable agreement with the present results, though the entrance region is rather less discernable in the results of Leung, Kays and Reynolds, than in the present results. The author knows of no other published work with which a comparison might be made.

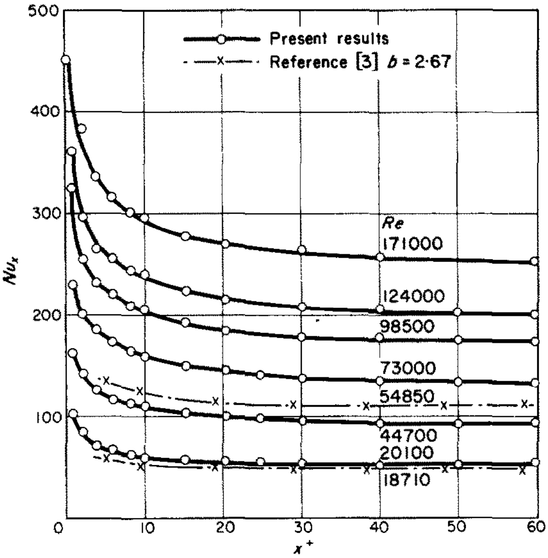

FIG. 8. Entrance region heat transfer, $b = 2{\cdot}88$.

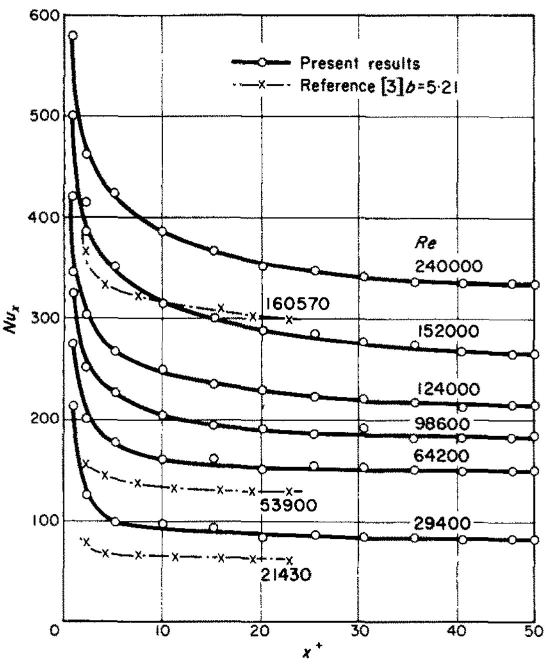

FIG. 9. Entrance region heat transfer, $b = 5{\cdot}67$.

The non-dimensional thermal entrance length x_e^+ in duct heat transfer has been variously defined. It is defined here as that length at which Nu_x has a value 5 per cent greater than Nu_{fd}. Experimental values were determined by drawing a straight line on the temperature plots parallel to the bulk temperature line and at the correct distance from it. Results are shown in Fig. 11 and compared with theoretical values for the parallel plate taken from the analysis by Hatton and Quarmby [8]. The agreement is fair. Also shown is the theoretical prediction for the circular tube given by Sparrow, Hallman and Siegel [9]. This shows rather less Reynolds number dependance than the present results and the values for x_e^+ are less. The theoretical results for the tube have not yet been confirmed by adequate experimental results.

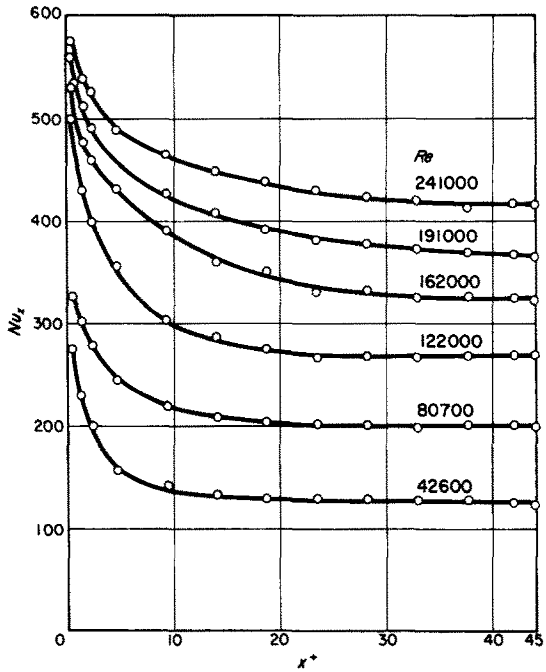

FIG. 10. Entrance region heat transfer, $b = 9{\cdot}14$.

The scatter in x_e^+ is due to the small size of the angle of intersection of the core temperature plot and the line drawn parallel to the bulk temperature.

DISCUSSION

The fluid properties in this work were evaluated at the local bulk mean temperature. This procedure correlated all the results for a given radius ratio on a single line as can be seen in

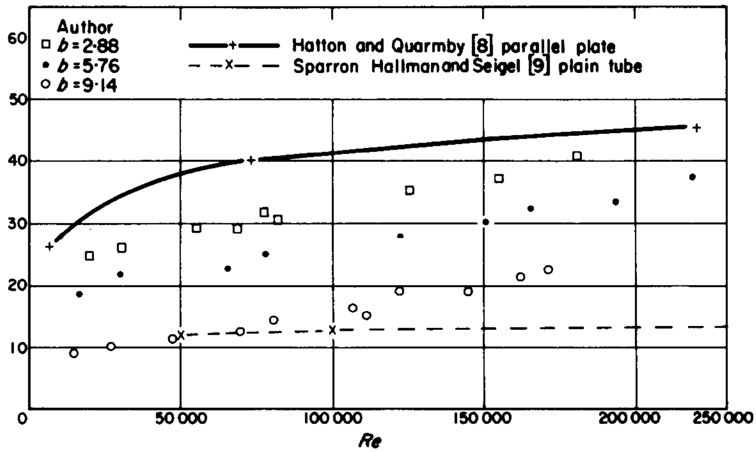

FIG. 11. Variation of thermal entry length with radius ratio and Reynolds number.

Fig. 6. The influence of variable fluid properties is thus clearly not significant for the present results. An estimate of whether variable fluid properties should be significant can be got from the ratio T_i/T_b. As stated the inner wall–bulk temperature differences in the present results were in the range 50–130 degF with most of the data lying in the lower part of the range. The absolute temperature ratios were correspondingly 1·09–1·22.

Deissler [10] and Deissler and Eian [11] have shown that the significant parameter in this respect is defined as

$$B = \frac{D}{4}\frac{1}{T_i}\frac{\mathrm{d}t_b}{\mathrm{d}x}\sqrt{\left(\frac{2}{f}\right)}.$$

Values of B in the present results were nowhere greater than 0·004. Comparison of this value and the temperature ratio range with the theoretical and experimental results of references [10, 11] entirely justifies correlating the present data by use of the bulk temperature.

According to the theoretical calculation of reference [11], a value of $B = 0{\cdot}01$ would reduce the Nusselt number by about 5 per cent. However, the experimental results for $0{\cdot}006 < B < 0{\cdot}009$ and for $0{\cdot}014 < B < 0{\cdot}015$ both seem to correlate on the same line and they plot between the theoretical lines for $B = 0$ and $B = 0{\cdot}01$. It would appear that the effect of variable fluid properties can certainly be neglected for $B < 0{\cdot}01$, although in reference [11], it is shown to be significant for $B = 0{\cdot}02$.

Leung, Kays and Reynolds [3], however, introduced a correction factor into their results which they said was to allow for the effect of variable fluid properties. This correction factor was arrived at by considering their results when the apparatus was run as a plain tube. The Nusselt number was calculated using the local bulk temperature and then multiplied by the ratio of the bulk temperature to the absolute wall temperature raised to the power 0·575. That is, the original results for the plain vertical tube were reduced to give agreement with the analysis of reference [9] and an empirical relation suggested by Kays [12], i.e.

$$Nu = 0{\cdot}023\,(Re)^{0{\cdot}8}\,(Pr)^{0{\cdot}6}.$$

Since this relation is not available in the general literature it is not possible to discover if a variable fluid properties correction was used in evaluating the experimental results on which it is based or not. However, it is very similar to the relation proposed by Dittus and Boelter [13], namely,

$$Nu = 0{\cdot}023\,(Re)^{0{\cdot}8}\,(Pr)^{0{\cdot}4}$$

where fluid properties were evaluated at the bulk temperature.

Several investigators reported by McAdams [14], notably Desmon and Sams [15] and references [10, 11] have given results for heat transfer from the heated wall of a plain tube under conditions such that the wall-bulk temperature ratio and B were large. When these results are correlated using the bulk temperature the effect of an increased value of T_w/T_b or B is to reduce the Nusselt number for a given Reynolds number. Thus, for $T_w/T_b = 3{\cdot}17$ and $Re = 80000$ reference [15] reports Nu as being less by about 70 per cent than for $T_w/T_b = 1{\cdot}6$.

Leung, Kays and Reynolds [3], however, as noted above, used a correction factor which reduced their Nusselt numbers to make them agree with an acceptable correlation based on the bulk temperature and moderate values of the wall–bulk temperature ratio. Thus, they were in fact suggesting an effect due to variable fluid properties which is quite opposite to that noted by previous investigators. Accordingly, the agreement between their results and the present results is surprising.

An empirical correlation was attempted for the fully developed heat-transfer result shown in Fig. 6. This was done by fitting a first-order polynomial by the method of least squares to the logarithms of each set of the Nu–Re data and averaging the exponents which differed by only 2 per cent. Similarly, the constants were expressed as a function of radius ratio. The result was

$$\log Nu = -K + 0{\cdot}706 \log Re \tag{4a}$$

and

$$\log K = 0{\cdot}1658 - 0{\cdot}1056 \log b \tag{4b}$$

which might be modified to include Pr as further data becomes available.

The correlation is shown on Fig. 6 and also on Fig. 7 as a function of b. It is interesting to note the way in which it extrapolates to the parallel plate case, $b = 1$, compared with the suggested extrapolation of reference [3]. The experimental point for the parallel plate case is taken from Barrow [16] but it is not very conclusive since the scatter present in those results was 10–15 per cent about the line of best fit from which this point is taken.

In conclusion, then, it is suggested that turbulent heat transfer to air from the core tube of a concentric annulus is described by equation (4) for Reynolds numbers up to 270000 and radius ratios less than about 10 if the temperature differences and heating rates involved are moderate. Also, the thermal entrance region decreases with radius ratio but increases with Reynolds number.

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Résumé—On présente les résultats de mesures du transport de chaleur turbulent à partir du tube central d'une conduite annulaire horizontale non excentrée. Les rapports des rayons étudiés sont 2,88; 5,76 et 9,14 avec une gamme de nombres de Reynolds de 5000 à 270000. L'étude couvre à la fois le régime entièrement établi et la région d'entrée thermique. Le profil de vitesse était entièrement établi au début du chauffage.

Quelques résultats sont donnés pour la variation de la longueur d'entrée thermique avec le nombre de Reynolds et le rapport des rayons.

Zusammenfassung—Es werden Messungen angegeben über den turbulenten Wärmeübergang vom Innenrohr einer waagerechten, konzentrischen Rohranordnung. Das Verhältnis der untersuchten Radien betrug 2,88, 5,76 und 9,14 im Reynolds-Zahlenbereich von 5000–270000. Die Untersuchung erstreckt sich sowohl auf den voll ausgebildeten Zustand als auch den thermischen Einlaufbereich.

Einige Ergebnisse sind wiedergegeben für den Einfluss der Reynolds-Zahl und des Halbmesserverhältnisses auf die thermische Einlauflänge.

Аннотация—Представлены результаты измерений турбулентного переноса тепла от внутренней цилиндрической стенки горизонтельного коаксиального зазора. Отношение радирсов стенок составляло: 2,88; 5,76 и 9,14, а диапазон значений критерия Рейнольдса 5000 - 270000. Изучался как полностью развитый процесс, так и термический входной унасток. В начале нагрева профиль скоростн был полмостчю стабилизированным.

С помощью критерия Рейнольдса и отношения радиусов представлены некоторые результаты по изменению длины термического входного участка.